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ABSTRACT

The purpose of this curriculum guide is to help the geometry teacher in his endeavor to fulfill his teaching responsibilities. Space is provided for teachers' additions, deletions, notes, and criticisms which will be useful when the guide is revised. The guide is arranged in vertical columns relating the geometry curriculum concepts to: curriculum performance objectives, career concepts and performance objectives, suggested teaching methods, and audio-visual and resource materials. An outline is included at the beginning of the guide connecting its topics with the geometry textbook used in the school district. Sources of audio-visual material are listed at the end. (DS)

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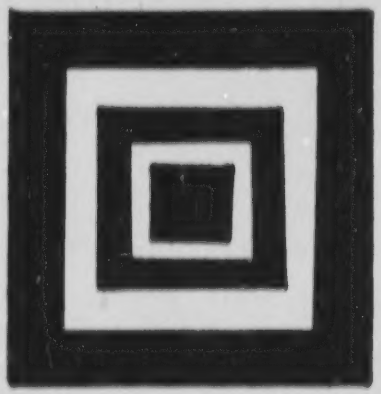

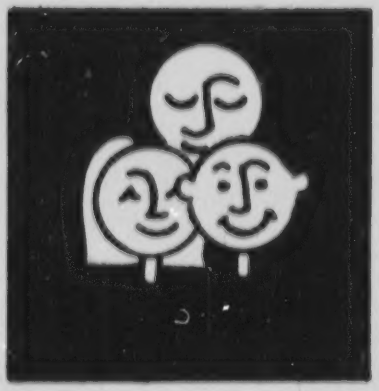

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Career - Curriculum Gu.

CAREER EDUCATION CENTER

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GEOMETRY

CE 001 009

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SAN ANTONIO, TEXAS

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GEOMETRY

CURRICULUM GUIDE

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A C K N O W L E D G E M E N T S

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Mr. William Thomas

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Mr. Hamilton C. Dupont - Head of Math Department

Mrs. Gozelle Loveless - Audio-Visual Coordinator

Mrs. Mikel A. Arnold - Teacher

Gratitude is also expressed to the Texas Education Agency, Character Education Project, Education Service Center-Region 20, Minnie Stevens Piper Foundation, and the Career Education Project Staff.

Preface

Meaningful existence is the goal of life in today's world. Living takes on meaning when it produces a sense of self-satisfaction. The primary task of education must be to provide each individual with skills necessary to reach his goal.

When children enter school, they bring with them natural inquisitiveness concerning the world around them. Normal curiosity can be the nucleus which links reality to formal training if it is properly developed. A sense of continuity must be established which places education in the correct perspective. Communities must become classrooms and teachers resource persons. Skills such as listening, problem solving, following directions, independent thinking and rational judgement then can merge into daily living procedures.

In classrooms especially designed to form a bridge between school and the world of work, experiences must be developed. On campus performance in job tasks and skills, following a planned sequence of onsite visitation, will fuse information into reality. Practical relationships developed with those outside the formal school setting will provide an invaluable carry-over of learned skills.

Search for a rewarding life vocation is never easy. Without preparation it becomes a game of chance. With a deliberate, sequential, and planned program of development, decisions can be made based upon informed and educated judgements.

A full range career education program, K-12, will offer opportunities for participants to enter employment immediately upon completion of training, post secondary vocational-technical education, and/or a four-year college career preparatory program.

C. N. Boggess

C. N. Boggess, Superintendent
Harlandale Independent School District

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PHILOSOPHY

The educational needs of any community are somewhat unique. This was certain to have been one of the guiding principles used when our forefathers set up local control for public schools. Accordingly, the philosophy of the Harlandale school system is to serve the educational needs of all its citizens as evidenced by adult classes, government sponsored retraining programs, vocational courses, etc. The mathematics department follows this philosophy in planning a program best suited to the needs of our students.

The past decade has proven the need for the new emphasis on the importance of the study of mathematics. Usefulness of mathematics in many fields of learning and endeavor, long thought to be free of mathematics, is now an accepted reality. Also, basic principles must be understood in order that mathematical systems can be devised to describe new human or mechanical activities, as they come to pass.

As mathematics continues to grow this must, of necessity, result in the addition of new symbols, terms, topics, and new approaches. The changing times will make some older topics and methods obsolete. To meet the mathematical needs of our students and to assist teachers in their instruction, the mathematics department has prepared this mathematics guide. Any given faculty consists of personnel with different degrees of training, experience, local tenor, and understanding of student needs. Hence, the desirability for some guiding criteria. In addition, we feel all students should consider career planning as a major facet of their education. Thus it follows that they will need some exposure to the different mathematical requirements of the varied career fields. In part, it is the purpose of this guide to assist the teacher in providing appropriate instruction to meet such needs.

The department feels that the most important basic guide for any mathematics course is the textbook, and careful care is taken in the selection of this book. Not only is the textbook an important guide for the teacher but it is also desirable for the student to learn the use of a textbook as a guide and important tool for learning.

Therefore, the plan of this mathematics guide is not to rewrite the textbook but to improve on it. Generally, the plan is to implement, where desirable, the textbook coverage, describe supplementary material that is needed and make suggestions on methods, procedures, order of coverage, etc.

Mathematics is a thoughtful, creative and intellectually stimulating subject. The enthusiasm and interest of the teacher in the subject is the best atmosphere for creating student enthusiasm for mathematics. This guide is planned to help foster this enthusiasm and in no way infringes on the academic freedom of the teacher.

It is hoped that the guide will prove helpful in understanding the basic standards, improving instruction, and developing the desired uniformity for the classes in each area of study. Finally, the guide should serve as the nucleus for a continuing effort to improve mathematics instruction.

Mr. Hamilton C. Dupont - Head of Math Department
Harlandale Independent School District

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Overview

The teaching of geometry has been undergoing change for as many as 60 years. Geometry is no longer an isolated course but is a basic part of the entire mathematics program. In elementary and middle school programs, it is treated informally, and the intuitive approach involves the identification and recognition of two-dimensional geometric figures as well as a study of their properties. Therefore, the high school course in geometry should emphasize something besides the acquisition of facts and skills in applying these facts to the practical world. The course which is to be taught in high school is a synthetic geometry developed as a mathematical system based on a postulational approach and written in a language that unifies concepts of geometry, of algebra, and of arithmetic by an effective use of the real number system.

Goals

To acquire a systematically organized body of geometric knowledge of physical space

To develop deductive thinking

To develop inductive and creative thinking

To gain insight into axiomatics and the construction of mathematical models

To master a variety of approaches

To relate geometric content to other branches of mathematics and prepare a foundation for further study of mathematics

To improve the student's understanding of the relationship between various careers and geometric concepts

The audio-visual materials listed in this guide have been assigned catalogue numbers by the Harlandale Independent School District audio-visual department or the Education Service Center, Region 20, San Antonio, Texas.

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The following outline is built upon a "conceptual ladder" (concepts from 'easiest to hardest') for geometry. The outline corresponds to the outline found in the curriculum concepts of this guide. The page numbers refer to the present textbook being used in geometry by the Harlandale Independent School District. (Jurgensen, Ray C., Modern School Mathematics Geometry, Boston: Houghton Mifflin Company, 1969.)

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CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
I. Basic Properties and Terminology A. History	I. THE STUDENT SHOULD BE ABLE TO: A. 1. Write a one or two page report on Euclid's contributions to mathematics. 2. Write a three to five page report on the Babylonian or Egyptian contributions to mathematics.	Concept Relationship of the history of geometry to the career of an archeologist. Performance Objective When given a rope or string with 12 equidistant knots, form a 3-4-5 right triangle as used by the ancient Egyptians for surveying purposes. General Information Students wishing extra activities should be encouraged to research other careers in archeology. They may wish to compile and analyze material done by archeologists. High school students wishing positions in archeology should obtain Ph. D. degrees. A master's degree, plus field experience, is sufficient for many beginning professional positions. In 1970 salaries ranged from $8,000 to $20,000 a year. Teaching Activity Archeologists excavate places where people lived in the past to reconstruct their history and customs by studying the remains of homes, tools, clothing ornaments, and other evidences of human life and activity. In their study of the ancient Egyptians an archeologist must be informed in geometry since much of Egyptian architecture depended on geometry. In excavating an ancient
B. Definitions	B. When given ten definitions, choose the five definitions which are acceptable out of the ten.	
C. Point, Length, Surface, Space, and Straight Line	C. Make a listing of undefined terms and give an example of each (point, length, surface, space, and straight line).	
D. Plane, Line Segment, Ray, Angle, and Adjacent Angle	D. Define with 80% accuracy a plane, line segment, ray, angle, and adjacent angles.	
E. Exterior and Interior of Angles	E. When presented with the drawing of an angle, distinguish the interior by shading it.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
I. A. It is helpful at the beginning of this course to allow the students to discover the former and present world of geometry. Point out how the study of geometry began by explaining how land was surveyed along the Nile River. Discuss the use of geometry today in such areas as architecture. Ask the students to make a list of objects which they recognize as geometric shapes (example: buildings). B. Explain to the students what is meant by a good definition. Give them examples of good and bad definitions. Ask the students on a written exercise to define words they already know. Let the students exchange papers and decide if the other person made good definitions. C. Point out that one needs undefined terms to be able to define further terms. Let them try to define the defined terms without using the undefined terms. D. Ask the students to give physical examples of a plane, line segment, ray, angle, and adjacent angle as they appear in the classroom. E. For practice have each student draw several angles with letters in the exterior, in the interior, and on the angle. Then have students exchange papers and make a listing of where each letter is located.	Curriculum Harlandale Audio-Visual Center Introduction; filmstrip -- X-27 Vocabulary Lines & Angles, I; filmstrip -- X-28 Vocabulary Lines & Angles, II; filmstrip -- X-29 Vocabulary Lines-Relationship; filmstrip -- X-30 ESC-Region 20 Introducing Shapes, Lines and Angles; 16mm film -- 4311 For Additional Information on Archeological Careers Archaeological Institute of America, 100 Washington Square East, New York, New York 10003 Smithsonian Institution, Washington, D.C. 20560. Society for American Archeology, 3700 Massachusetts Avenue, N.W., Washington, D.C. 20016.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
F. Geometric Solids 1. Definition	F. 1. Define a geometric solid as a portion of space completely separated from the rest of space by some kind of surface or surfaces.	Egyptian community it is helpful to the archeologist to know the general lay-out of the village before digging. With his knowledge of geometry the archeologist knows that the Egyptians surveyed their land by using a rope with 12 equidistant knots which formed a 3-4-5 right triangle. By applying this principle the archeologist can greatly improve his chances of making an important find before excavating.
2. Polyhedrons	2. Verify Euler's formula (V-E+F = 2; V = number of vertices, E = number of edges, and F = number of faces) for a cube and a pyramid.	
G. Congruence	G. Given six geometric figures, identify four which are congruent.	
H. Angle Measurement	H. Given ten angles, choose four which are the same measure but in different positions.	
I. Classification of Angles	I. Define straight angle, perpendicular lines, right angle, obtuse angle, supplementary angles, and complementary angles.	

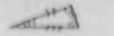
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
F. 1. Show as many models of geometric solids as possible. For extra credit ask the students to make models out of construction paper or plastic. 2. Point out the vertices, edges, and faces of a model of a pyramid. Ask a student to check the validity of Euler's formula by showing V-E+F = 2 for a pyramid. G. Have the students make a listing of objects which they know are congruent. Make sure that they do not mistake objects that 'look alike' for congruent objects. H. If possible, use a plastic protractor on the overhead projector to explain angle measurement. Make sure each student has a protractor. For exercises you might duplicate angles to be measured. It is also a good idea to place some of the angles to be measured in different orientations, so that the student gets practice using the protractor upside down or in any position. I. Ask the students to draw three examples of straight angles, perpendicular lines, right angles, obtuse angles, supplementary angles, and complementary angles. Stress that only two angles make up supplementary or complementary angles.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
II. Reasoning and Direct Proof A. Need for Proof	II. THE STUDENT SHOULD BE ABLE TO: A. Bring one example of an optical illusion to class.	Concept Relationship between using deductive reasoning and the job of a research scientist. Performance Objective Name one way in which the research scientist uses deductive reasoning in performing his job. General Information Students wishing extra activities should be encouraged to research other careers in scientific work. They may wish to compile and analyze material on work done by the scientist. Approximately 472,000 scientistswere employed in the United States in 1970. Salaries in the scientific field vary according to degree and experience. In order to begin a scientific career a student should expect to receive at least a bacheler's degree with a major in one of the fields of science. Teaching Activity The scientist must often use deductive reasoning in his research. In order to use deductive reasoning he must collect and organize established fact from which he can reason a valid conclusion. As an example he may use the following outline of the scientific method to accomplish his result. I. Define the problem. II. Collect information that relates to the problem.
B. Methods of Reasoning	B. When given ten assertations, determine four which depend on inductive reasoning and six which depend on deductive reasoning.	
C. Logical Proof	C. Complete a truth table which specifies the conditions under which a conjunction is a true statement.	
D. "If-Then" Pattern	D. When given five "If-Then" statements, list the hypothesis and conclusion for each.	
E. Postulate	E. Define a postulate as an arbitrary statement that forms the foundation for the structure of a particular mathematical model which one plans to develop.	
F. Equality	F. Given ten statements of equality, determine with 80% accuracy which property of equality applies (reflexive, symmetric, or transitive).	
G. Points and Lines	G. Give two physical examples of two intersecting lines determining a point.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
II. A. Illustrate by example how students use proof in their everyday life. Point out that things may not always be as they appear by using optical illusions. B. Explain the difference between induction, deduction, and intuition by numerous examples. Ask the students to write two examples of each way of reasoning. C. Using algebraic examples will aid the student in understanding the basic set up of a proof. Give the students ten algebraic problems and have them solve the problems in proof form. D. Use several sentence examples as well as algebraic examples to show what the two parts of an "If-Then" are. Example: If $\frac{1}{4}X = 16$; Then $X = 64$. The first statement is the hypothesis which leads to the second part which is the conclusion. E. A postulate should be stressed as being a statement accepted as true and requiring no proof. Give examples of postulates we accept in everyday life such as "today is Monday." Have the students make a list of ten postulates which they know are true. F. Stress the use, by examples, of the symmetric, reflexive, and transitive properties. G. Ask the students to list examples in the classroom of two lines intersecting to form a point such as two pieces of molding that meet at a corner of the room. Have each student list at least ten examples.	Curriculum Harlandale Audio-Visual Center Logic Deductive Reasoning; filmstrip -- X-40 Logic Definitions; filmstrip; -- X-39 Logic Introduction; filmstrip -- X-41 Logic Mistakes in Thinking; filmstrip -- X-42 ESC-Region 20 Geometry-Inductive and Deductive Reasoning; 16mm film -- 8155 Career Harlandale Audio-Visual Center Research Scientist; cassette tape -- cas. T-34 For Additional Information on Scientific Careers American Astronomical Society, 211 Fitz Randolph Rd., Princeton, N. J. 08540. American Chemical Society, 1155 16th St. NW., Washington, D.C. 20036.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
H. Midpoint	H. Define a midpoint as a point which divides a geometric figure into two congruent parts.	III. Form the hypothesis. IV. Test the hypothesis. V. Accept or reject the hypothesis. VI. Communicate the results.
I. Planes 1. Determination 2. Intersection	I. 1. Name the smallest number of points which determine a plane. 2. Given a three dimensional drawing of the intersection of two planes, name the end points of the line segment which is the intersection.	
J. Angles	J. Define an angle bisector as a line segment which divides an angle into two equal parts.	
K. Theorem	K. Define a theorem as a statement that can be proved.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
H. Stress the prefix mid and show how it follows from middle. Also stress that it is one point (not a line). I. 1. Stess that a plane is determined by three points not in the same straight line. (example: tripod) 2. Use a physical model to illustrate the intersection of two planes. Visual examples are very important. J. When covering the definition of an angle bisector compare it with a midpoint. A midpoint and a bisector both cut something into two congruent parts. For practice on the protractor have the students draw five angles and bisect each. K. Ask the students to work five algebraic problems showing that a theorem can be proved. This exercise gives the students a chance to review their algebra.	American Geological Institute, 2201 M St. NW., Washington, D.C. 20037. American Institute of Biological Sciences, 3900 Wisconsin Ave. NW., Washington, D.C. 20016. American Institute of Physics, 335 East 45th St., New York, N.Y. 10017. Interagency Board of U.S. Civil Service Examiners for Washington, D.C., 1900 E. St. NW., Washington, D.C. 20415. Printing and Publishing Office, National Academy of Sciences, 21101 Constitution Ave. NW., Washington, D.C. 20418. Society of Exploration Geophysicists, P.O. Box 3098, Tulsa, Okla. 74101.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
III. Triangle Relationships A. Properties 1. Rigidity 2. Naming Parts B. Classification 1. Sides 2. Angles C. Perimeter and Length Relationships D. Congruency	III. THE STUDENT SHOULD BE ABLE TO: A. 1. Name two places where triangles are used in architecture to make a structure rigid. 2. When given ΔABC, name the vertices and the sides of the triangle using the A, B, and C. B. 1. a. Define a scalene triangle as a three sided polygon with no two sides congruent. b. Define an isosceles triangle as a three sided polygon with at least two congruent sides. c. Define an equilateral triangle as a three sided polygon with all sides congruent. 2. Define an acute triangle as a triangle having each of its angles less than 90^0. C. 1. Define the perimeter of a triangle as the sum of the lengths of the sides of the triangle. 2. Prove with 80% accuracy the theorem which states the sum of the lengths of any two sides of a triangle is greater than the length of the third side. D. 1. Define the symbol "≅" to mean congruent. 2. When given ten sets of congruent triangles, mark corresponding parts which are equal (example: ).	Concept Relationship of the rigidity of triangles to the job of a civil engineer. Performance Objective When given five different plane geometric shapes, choose the triangle as the most rigid. General Information Students wishing extra activities should be encouraged to research other careers in engineering. They may wish to compile and analyze material on work done in engineering. Examples: 1. Aerospace Engineers 2. Agricultural Engineers 3. Biomedical Engineers 4. Ceramic Engineers 5. Chemical Engineers 6. Civil Engineers 7. Electrical Engineers 8. Industrial Engineers 9. Mechanical Engineers 10. Metallurgical Engineers 11. Mining Engineers Employment opportunities are expected to increase very rapidly through the 1970's. Salaries for engineers in 1970 ranged from a $10,400 average for engineers with a bachelor's degree to a $16,000 average for engineers with a Ph.D. High school students wishing to enter engineering should obtain a bachelor's degree in engineering although some

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
III. A. 1. Give examples of the rigidity of triangles by using models made of wood or plastic. 2. Draw a triangle on the board with the vertices labeled with an A, B, and C. Name one angle and one side of the triangle by using the letters. Ask the students to name the remaining angles and sides. B. 1. Make certain at this point that the students learn the difference between scalene, equilateral, and isosceles triangles. Ask the students to make a drawing of each of the triangles. 2. In addition to identifying an acute triangle it would be helpful to define an obtuse triangle and a right triangle. Stress that a so called "regular triangle" does not exist. Ask the students to draw three of each type of triangle. C. Ask the students to draw five triangles and measure the perimeter of each. When measuring the triangles, ask the students to show that the sum of the measures of any two sides is greater than the third side. D. Stress that congruency and equality mean the same thing. After explaining how to list corresponding parts of congruent triangles, show how to list the parts just by looking at the names of the triangles. If triangle ABC is congruent to triangle FGH then AB is congruent to FH, angles A & F, B & G, and G & H are congruent.	Curriculum Harlandale Audio-Visual Center Congruent Figures; filmstrip -- AA-77 Congruent Triangles; filmstrip -- Z-53 Postulates Triangles & Circles, filmstrip -- X-36 Vocabulary Triangles; filmstrip -- X-31 ESC-Region 20 Triangles-Types and Uses; 16mm film -- 4590 Career Harlandale Audio-Visual Center Civil Engineer; cassette tape -- cas. T-54 Your Future in Engineering Technology; record with filmstrip -- PR-786 For Additional Information on Engineering Careers Engineering Manpower Commission, Engineers Joint Council, 345 East 47th St., New York, N.Y. 10017. Engineers' Council for Professional Development, 345 East 47th St., New York, N.Y. 10017.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
E. S.A.S.	E. When given ten sets of triangles with three corresponding parts on each set marked congruent, determine with 90% accuracy from the given information which triangles fit the congruence S.A.S.	persons without a degree may become engineers after long experience in a related occupation -- and some college training. The student of engineering needs a thorough background in math and science. Teaching Activity A civil engineer is faced with designing a structurally sound bridge with no more than 20 tons of steel. In working with several designs he discovers that a bridge with a rectangular framework and the desired weight load would require 32 tons of steel. The same bridge with a triangular framework would require 19 tons of steel. If possible, use geometric models to demonstrate the rigidity of the triangle.
F. A.S.A.	F. When given ten sets of triangles with three corresponding parts on each set marked congruent, determine with 90% accuracy from the given information which triangles fit the congruence A.S.A.	
G. Overlapping	G. When given a drawing of overlapping congruent triangles, make a listing with 90% accuracy of the congruent parts.	
H. Auxiliary Line	H. Define an auxiliary line as a line introduced to make a proof of a theorem possible.	
I. Angles of an Isosceles Triangle	I. Prove with 80% accuracy the theorem which states the angles opposite the equal sides of a triangle are equal.	
J. Corollary	J. Define a corollary as a statement that is readily proved by applying a theorem.	

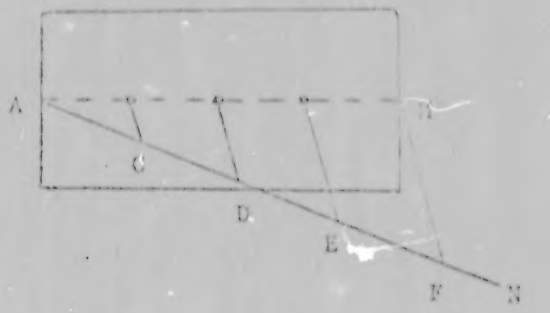
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
E. In explaining the congruency S.A.S. stress that the angle must be the included angle. Ask the students to draw several triangles and list the parts which must be congruent if the triangles are congruent by S.A.S. F. In explaining the congruency A.S.A. stress that the side must be the included side between two angles. At this point ask the students to draw five triangles each which shows S.A.S. and A.S.A. G. Overlapping can be a difficult concept for some students. Suggest that the students draw overlapping triangles separately. For practice have the students draw some of the overlapping triangles that are given for proof problems separately. H. Stress that a student should not draw an auxiliary line any time he thinks it is necessary. It is also important to note that an auxiliary line should be shown distinctly from the original diagram by using a different colored pencil or dashed lines. I. The students should be impressed with the knowledge that the base angles of an isosceles triangle are congruent. Illustrate physical proof by asking the students to draw five triangles with two congruent sides. Then ask the students to measure the base angles to see if they are congruent. J. Point out that a corollary is closely linked to the theorem it follows. Show the proof of a corollary by applying a previously proved theorem.	National Society of Professional Engineers, 2029 K St., NW., Washington, D.C. 20006. The American Federation of Technical Engineers (AFL-CIO), 1126 16th St. NW., Washington, D.C. 20036.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
K. Statements 1. Converse	K. 1. Define converse as the interchange of all or part of the hypothesis with all or part of the conclusion of a theorem.	
2. Inverse	2. When given a theorem, write its inverse by writing the negation of the hypothesis and conclusion.	
3. Contra-positive	3. Define contrapositive as the inverse of the converse or the converse of the inverse.	
L. S.S.S.	L. Prove the theorem with 80% accuracy that states if three sides of one triangle are equal, respectively, to three sides of another triangle, then the triangles are congruent.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
K. 1. A converse may be shown by the use of a truth table. Give sentence examples as well as truth table examples. example: (statement: All right angles are equal to 90°. converse: All 90° angles are right angles.) 2. An inverse may be shown by the use of a truth table. If a statement is true then the inverse of it is false. 3. A contrapositive may be shown effectively by the use of a truth table. Ask the students to work out both two and three part truth tables for contrapositives. L. Emphasize that two triangles are congruent only if the corresponding sides are congruent. Remind the students how to list two congruent triangles in the correct order depending on corresponding parts. For practice list five pairs of triangles that are congruent and ask the students to list all of the corresponding parts.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
IV. Constructions A. Circle	IV. THE STUDENT SHOULD BE ABLE TO: A. Define a circle as the set of points in a plane that are at a given distance from a given point in the plane called the center.	Concept Relationship of geometric constructions to the job of the draftsman. Performance Objective Given the correct dimensions, locate by constructing parallel lines the points at which three holes should be drilled on a gasket template. General Information Students wishing extra activities should be encouraged to research drafting as a career. They may wish to compile and analyze material on work done in drafting. Approximately 310,000 draftsmen were employed in 1970; almost 4 percent were women. High school students wishing to enter drafting can obtain training from a number of sources (technical institutions, junior and community colleges, vocational and technical high schools, correspondence schools, on-the-job training programs, and 3 or 4 year apprenticeship programs). The average salary for a draftsman was $700 per month in 1970. Employment opportunities for draftsmen are expected to be favorable through the 1970's. Prospects will be best for those having post-high school drafting training.
B. Definition	B. Define a construction as an imaginary drawing in which required points or lines are determined by use of a compass and a straightedge.	
C. Types 1. Bisecting an Angle	C. 1. When given ten angles, bisect eight out of the ten angles correctly.	
2. Perpendicular	2. When given ten line segments with a specified point on each line, construct a perpendicular to the given line at the given point with 80% accuracy.	
3. Bisecting a Line Segment	3. When given ten line segments, bisect eight out of the ten line segments correctly.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
IV. A. Although most students have had some experience in the use of the compass, it is helpful at this point to instruct the students on how to draw a circle using a compass. Ask the students to practice drawing circles using unequal radii. B. Stress that in geometric construction one may use compass and straightedge only. Emphasize that the compass and straightedge serve as measuring instruments. Students should not use a ruler or a protractor except to check for accuracy. C. 1. It is helpful for the teacher to check angle bisections made by the students with a protractor. The student should also be encouraged to check his work with a protractor as long as the protractor is not used in the original construction. 2. After a teacher explanation, ask several students to construct at the board a perpendicular to a point on a line and also a perpendicular from a point not on a line to a line. Each student in the class should practice the constructions at their desk. 3. Ask each student to draw five line segments and practice bisecting them. Check for accuracy with a ruler. It may be helpful to show how to construct a perpendicular bisector and emphasize that a bisector does not have to be perpendicular.	Curriculum Harlandale Audio-Visual Center *Bisecting Angles and Segments*; filmstrip -- AA-78 Career Harlandale Audio-Visual Center *Your Future as a Draftsman*; magnetic tape -- MJ-258 For Additional Information on Drafting Careers American Federation of Technical Engineers, 1126 16th Street, NW., Washington, D.C. 20036. American Institute for Design and Drafting, Post Office Box 2955, Tulsa, Oklahoma 74101.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
4. Congruent Angles	4. When given ten angles, construct a congruent angle beside each angle with 80% accuracy.	Teaching Activity A draftsman is a person who draws plans and blueprints of machines, houses, automobiles and almost anything that can be manufactured. These plans must be precise in their dimensions, and are usually drawn to scale. Suppose you are a draftsman and are working for a company which makes air compressors. A blueprint for a gasket is needed with 3 equally spaced holes along a center line. The blueprint is 4 3/16 inches long and 3 inches wide. Using parallel lines, locate the points at which the 3 holes should be drilled. Draw line AN at an angle to AM. Use a compass to mark off 4 equal segments on line AN. Mark points CDEF. Draw a line from F to M. Using the compass, draw lines through C, D, E, and F parallel to FM. The intersection of lines with AM will locate the centers of the 3 equally spaced holes.
5. Triangles	5. a. When given ten sets of three line segments, construct triangles from the given segments with 90% accuracy. b. When given one line segment and two angles or two line segments and one angle in ten exercises, construct triangles from the given information with 80% accuracy.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
4. Accuracy of work is most important in constructing congruent angles. Each student should draw two acute, two obtuse, and two reflex angles. The student should then construct congruent angles to each of the angles he has drawn. 5. Constructing triangles is a useful way of reviewing ways to prove that two triangles are congruent. a. Ask the students to draw three unequal line segments and construct a triangle using the line segments. If the lines do not make a triangle explain to the student that the sum of any two sides have to be greater than the other two sides. b. Explain the construction of triangles by using one side and two angles or by using one angle and two sides. Ask the students to make an exact copy of these constructions at their desk using their own construction instruments.		

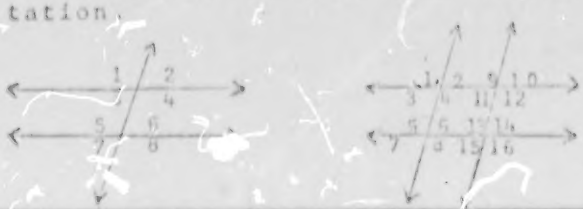
CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
V. Perpendicular Lines and Planes A. Lines on a Plane	V. THE STUDENT SHOULD BE ABLE TO: A. State that there exists only one line perpendicular to a given line through a point on the line.	Concept Relationship of vertical and horizontal lines to the job of a surveyor. Performance Objective When given a plumb line, determine if the door facing in the classroom is plumb (vertical). General Information Students wishing extra activities should be encouraged to research other careers in surveying. They may wish to compile and analyze material on work done by surveyors. Surveying is a limited occupation in regard to number of jobs available. There are approximately 50,000 licensed surveyors in the United States. The average salary for surveyors in the United States in 1970 was $8,850 per year. The most common method of preparing for work as a surveyor is through a combination of post-secondary school courses in surveying and extensive on-the-job training in survey techniques and in the use of survey instruments. The entrance requirement for most surveying programs is high school graduation (preferably including courses in algebra, geometry, trigonometry, calculus, drafting, and mechanical drawing). Teaching Activity A surveyor must often align his
B. Theorem	B. Prove the theorem with 80% accuracy that states in a given plane only one perpendicular can be drawn to a given line through a point not on the line.	
C. Points on a Perpendicular Bisector	C. Prove the theorem with 80% accuracy which states that any point on the perpendicular bisector of a line segment is equidistant from the ends of the segment.	
D. Dimensions	D. Draw a three-dimensional frontal view of a vertical line, horizontal line, vertical plane, and horizontal plane.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
V. A. Emphasize that two lines in a plane are either parallel or intersecting and that two intersecting lines are either perpendicular or oblique. Have each student draw three pairs of parallel lines, three pairs of perpendicular lines, and three pairs of oblique lines. Show examples by use of models. B. Ask a student to go to the board and draw more than one line perpendicular from a point not on the line to the line. Try several examples and measure the angles with a protractor if necessary. C. Give a formal presentation of the theorem which states in a given plane only one perpendicular can be drawn to a given line through a point not on the line. After proving this theorem have each student construct a perpendicular bisector of three different lines. Then by use of a ruler ask the students to measure in order to determine if each point on the perpendicular bisector is equal distant from the ends of the line. D. If available, use models to present a view of a vertical line, horizontal line, vertical plane, and horizontal plane. For a written exercise ask the students to make a three-dimensional frontal view of each of these geometric figures.	Career Harlandale Audio-Visual Center Your Future as a Surveyor; magnetic tape -- MT-270 ESC-Region 20 Measuring in Astronomy-How Big, How Far; 16mm film -- 8839 For Additional Information on Surveying Careers American Congress on Surveying and Mapping, Woodward Building, 733 15th St. NW., Washington, D.C. 20005. American Society of Civil Engineers, 345 East 47th Street, New York, New York 10017. American Society of Photogrametry, 105 North Virginia Ave., Falls Church, Va. 22046.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CATALOG CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
E. Line Perpendicular to a Plane	E. Given the necessary materials, construct a model of a line perpendicular to a plane.	equipment vertically in order to perform his job. A vertical line at any point on the earth's surface is the line which follows the direction of gravity at that point. It is the direction which a string will assume if a weight is attached to the string and the string is suspended freely at the point. In order for the students to see an application of a plumb line use a string and weight to check the vertical accuracy of the door facing in the classroom.
F. Perpendicular Planes	F. Given the necessary materials, construct a model of two perpendicular planes.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
E. For extra credit have students build a model of a line perpendicular to a plane. This model may be constructed from a variety of materials such as wood or plastic. F. If possible, use a model to present perpendicular planes. Also explain the meaning of a dihedral angle which may be illustrated by an open book.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
VI. Indirect Proof and Parallel Line Relations	VI. THE STUDENT SHOULD BE ABLE TO:	
A. Indirect Proof	A. Define an indirect proof as a method of proof which assumes the proposition false and then shows that the assumption is absurd.	
1. Contradictory Statement	1. Define a contradictory statement as any statement denying the truth of another statement.	
2. Format-Sample	2. Prove by indirect proof with 80% accuracy the theorem which states if two lines are perpendicular to a given line, all coplanar lines, then, the lines are parallel.	
B. Parallel Postulate	B. Make a drawing which illustrates Euclid's parallel postulate. "Through a point outside a line not more than one parallel can be drawn to the line."	
C. Parallel Lines and Angle Relationships	C.	
1. Transversals	1. Make a drawing of a transversal illustrating a line intersecting two or more lines in different points and label with 80% accuracy corresponding angles, alternate interior angles, and supplementary angles.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
VI. A. It may be helpful to discuss with students how they use a method of indirect proof in everyday life. When students take a multiple choice test and eliminate all the choices except one they are using indirect proof. 1. Ask each student to write ten statements. Next ask the students to exchange papers and write ten statements which contradict the first ten. 2. Ask each student to try to draw two lines perpendicular to the same line that will not be parallel. They will discover that this is possible only if the two lines are in different planes. B. Discuss with the class how the parallel postulate and other theorems about parallel lines are used in sports, carpentry, electricity, and map making. Ask each student to list as many examples as he can. C. 1. At this point the student should be made familiar with corresponding angles, alternate interior angles, and supplementary angles in regard to parallel lines cut by a transversal. Use diagrams to make this presentation. 	Curriculum Harlandale Audio-Visual Center The Parallel Postulate; filmstrip -- Z-50 Career Harlandale Audio-Visual Center Metalworking Series; records with filmstrips -- PR-737(BB-76) through PR-738(BB-79) For Additional Information on Ironworking Careers Associated General Contractors of America, Inc., 1957 E. St. NW., Washington, D.C. 20006. International Association of Bridge, Structural and Ornamental Iron Workers, 3615 Olive Street, St. Louis, Missouri 63108.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CURRICULUM CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
2. Contrapositive Statements	2. Make a Venn diagram to illustrate a contrapositive in regard to parallel lines.	Teaching Activity The ornamental ironworker often constructs grillwork which is composed of parallel bars which are held in place by two transversal rods. In constructing the grillwork he must constantly check to see that the bars are parallel. One method of checking the parallel bars is by seeing if the alternate interior angles are equal. By using a carpenter's square he can check to see that the alternate interior angles are equal to 90^0. Illustration: $\angle 1 = \angle 2$ $\angle 3 = \angle 4$ $\angle 5 = \angle 6$ All angles are 90^0 as checked by the carpenter's square.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
2. It is suggested that algebraic examples first be used in explaining contrapositive statements. a. Statement: $(x+5=0)\rightarrow(x\neq1)$ Contrapositive: $(x=1)\rightarrow(x+5\neq0)$ b. Statement: $(x^2-8x+15=0)\rightarrow(x\neq20)$ Contrapositive: $(x=20)\rightarrow(x^2-8x+15\neq0)$ c. Statement: Two lines are parallel if they are perpendicular to the same line. Contrapositive: If two lines are not perpendicular to the same line, then they are not parallel.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES
VII. Polygons	VII. THE STUDENT SHOULD BE ABLE TO:
A. Definition	A. Define a polygon as a simple closed figure formed by line segments.
B. Classification	B. Draw and label polygons for 3 through 12 sides.
C. Sum of Angles	C.
1. Triangle	1. Demonstrate by measuring the angles of at least three different triangles that the sum of the angles of any plane triangle is 180°.
2. Other Polygons	2. Find the sum of the angles of polygons having 4 to 20 sides with 80% accuracy by using the formula (n-2) 180°.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
VII. A. Ask each student to draw two convex and two concave polygons. B. Ask each student to draw and label a triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon, undecagon, and dodecagon. C. 1. Ask each student to draw an acute triangle, an obtuse triangle, and a right triangle. Then ask the students to measure the angles of each triangle and see if they add up to 180°. As an extra credit project ask the students to find examples of triangles whose angles add up to more than 180°. 2. Ask each student to draw five polygons which have different numbers of sides. Ask each student to compare the answer they obtain by using the formula $(n-2)180^{\circ}$ with the answer they obtain when they measure the angles to find the sum.	Curriculum Harlandale Audio-Visual Center Segments and Polygons; filmstrip -- AA-76 Vocabulary Polygons; filmstrip -- X-32	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CURRICULUM CONCEPTS, PERFORMANCE OBJECTIVES, ORIGINAL INTRODUCTION AND TEACHING SUGGESTIONS
		of the sum of the angles should aid in the design. Since the sign should be a regular octagon each angle must be equal. Solution: $S = (n-2)180^\circ$ $S = (8-2)180^\circ$ $S = 6 \cdot 180^\circ$ $S = 1080^\circ$ = sum of the angles of an octagon $\frac{1080^\circ}{8} = 135^\circ$ for each of the eight angles

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES
VIII. Quadrilaterals A. Definitions	VIII. THE STUDENT SHOULD BE ABLE TO: A. When given drawings of a parallelogram, rhombus, rectangle, and square; label each figure with its correct title.
B. Symbol	B. Define the symbol ▱ to mean parallelogram.
C. Trapezoid	C. Define a trapezoid as a quadrilateral with two of its sides parallel.
D. Project	D. Construct a parallel ruler as a special project.
E. Constructions	E. Given the line segments for the sides, construct a square and a rectangle.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
VIII. A. Define the parallelogram, rhombus, rectangle, and square in that order. Be sure to stress that the latter three are all parallelograms. Ask each student to draw and label a diagram of each parallelogram. B. Stress that only the parallelogram may be abbreviated by the use of a small parallelogram (no such symbols should be used for any other quadrilaterals). C. Explain the difference between a trapezoid and a parallelogram. Ask each student to draw one trapezoid with its parallel sides vertical and one trapezoid with its parallel sides horizontal. D. For an extra credit project ask a student to construct a workable model of a parallel ruler. Ask the student to make a demonstration of the instrument at the board. E. Select students from the room to go to the board and draw line segments from which squares and rectangles may be constructed. Ask each student at the board to select a student in the room to perform the construction by using the given line segments.	Curriculum Harlandale Audio-Visual Center Parallelograms and Their Properties; filmstrip -- Z-49 Career Harlandale Audio-Visual Center Forester; cassette tape -- cas. T-46 Lumberman, The; 16mm film -- 16-473 For Additional Information on Lumbering Careers American Forest Institute, 1835 K Street NW., Washington, D.C. 20006. American Forestry Association, 919 Seventeenth Street NW., Washington, D.C. 20006.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES
IX. Inequalities A. Definition	IX. THE STUDENT SHOULD BE ABLE TO: A. Define an inequality as a mathematical sentence using one of the following order symbols such as: $\neq$, $>$, $<$, $\geq$, or $\leq$.
B. Symbols	B. Mark true or false with 80% accuracy mathematical sentences which contain the symbols of inequality.
C. Axioms of Inequality	C. Use the axioms of inequality to solve inequalities in one variable with 90% accuracy.
D. Inequalities in Geometry	D. Prove the theorem with 80% accuracy which states, "The exterior angle of a triangle is greater than either opposite interior angles."

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
IX. A. Ask each student to make a sentence example using each of the inequality symbols. Then ask them to repeat the statements leaving out the symbols of inequality and request another student to fill in the correct symbols. Continue this process until the students are accurate in their work. B. Ask each student to make numerical examples using each of the symbols for inequality. Then let the students exchange papers and try to supply the correct symbol. C. Review using the addition, subtraction, multiplication, division, transitive, and powers and roots axioms concerning inequalities and then ask each student to work numerous problems until they show that they understand the use of the axioms. The problems should be mainly algebraic problems using one variable in orier that one may easily understand the axiom being used. D. Present the students with several problems using numbers and have them prove (using these numbers) that any two sides of a triangle when added together are greater than the third side. Also ask them to prove numerically that the exterior angle of a triangle is larger than either of the remote interior angles.	Curriculum Harlandale Audio-Visual Center Equations & Inequalities; filmstrip -- Z-30 Solving Inequalities; filmstrip -- Z-37 For Additional Information on Economic Careers American Economic Association, 1313 21st Avenue South, Nashville, Tenn. 37212. American Economic Association, 629 Noyes Street, Evanston, Illinois 60201. Joint Council on Economic Education, 1212 Avenue of the Americas, New York, New York 10036.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		manufacture any number of AM models up to (and including) 400 per month or any number of AM-FM models up to 300 per month. It takes 10 man-hours to produce an AM model, and 15 man-hours to produce an AM-FM model. The firm has up to 5,000 man-hours available for production of the two radios each month. If the profit gained on each AM model is $5 and on each AM-FM model is $7, find the number of each kind of radio the firm should manufacture to gain the maximum profit each month. The firm has a full time economist to solve problems of this nature. In solving the problem the economist may let x represent the number of AM radios and y represent the number of AM-FM radios which are manufactured each month. Therefore, $5x$ = monthly profit on AM models $7x$ = monthly profit on AM-FM models $5x + 7x$ = combined profit on both models Constraints of production: x and $y \geq 0$ (company must produce both models); $x \leq 400$ (greatest number of AM models company can produce each month); $y \leq 300$ (greatest number of AM-FM models company can produce each month); $10x + 15y \leq 5{,}000$ (5,000 man-hours are to be used for production)

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		Graphic solution of this problem gives corner points of (0, 0); (400, 0); (400, 66); (50, 300); and (0, 300). Using these values in the expression 5x + 7y the economist finds a maximum production level and a maximum profit for (400, 66). Therefore, the manufacturer should produce 400 AM models and 66 AM-FM models for a maximum profit of $2,462.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
X. Lines and Planes in Space A. Definitions 1. Skew Lines	X. THE STUDENT SHOULD BE ABLE TO: A. 1. Define skew lines as two lines which do not lie in any one plane.	Concept Relationship of the study of parallel lines to the job of an apparel designer. Performance Objective Given a parallel ruler or pantograph, reduce the size of an apparel drawing. General Information Students wishing extra activities should be encouraged to research other careers in the apparel industry. They may wish to compile and analyze material on work done in the apparel industry. Examples: 1. Designing room occupations 2. Sample stitcher 3. Patternmaker 4. Cutting room occupations 5. Hand spreader 6. Marker 7. Cutter 8. Sewing room occupations 9. Tailoring occupations 10. Pressing occupations 11. Fur shop occupations Designers are usually required to have considerable artistic ability as well as the talent of the draftsman. About 1.4 million persons were employed by the apparel industry in 1970. The average salary for all workers in the apparel industry was $84.37 a week in 1970. On the other hand beginning
2. Line Parallel to a Plane	2. Define a line parallel to a plane as a line and a plane that have no point in common.	
3. Parallel Planes	3. Define parallel planes as planes that have no point in common.	
4. Distance between Parallel Planes	4. Define the distance between two parallel planes as the perpendicular distance between two planes.	
B. Postulates	B. Build as a special project a three-dimensional model of one of the following: (1) skew lines, (2) line parallel to a plane, or (3) parallel planes.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
X. A. 1. Ask each student to make a list of ten pairs of lines that he thinks are skew lines. The student should be able to get these by examining the classroom. After they have completed their listings discuss the various answers and determine which ones are correct. 2. If possible, use a model. The teacher may use a sheet of paper and a pencil. Show how these two will never intersect if they are parallel. 3. The teacher may show parallel planes by using two pieces of paper that are not in the same plane. Ask each student to draw three pairs of parallel planes. 4. Show by the use of two pieces of paper or similar objects that when one wants to measure the distance between two parallel planes that the perpendicular distance is used. Show with a ruler that the perpendicular distance is the shortest distance. Ask each student to measure the distance between two parallel planes first not using a perpendicular line, then using a perpendicular line. B. Ask each student to draw: 1. Two skew lines, 2. A line or several lines parallel to a plane, 3. Two parallel planes, and 4. The distance between two parallel planes.	For Additional Information on Apparel Designing Careers American Apparel Manufacturers Association, Inc., 2000 K St. NW., Washington, D.C. 20006. Clothing Manufacturers Association of U.S.A., 135 West 50th St., New York, N.Y. 10020. International Association of Clothing Designers, 12 South 12th Street, Philadelphia, Pa. 19107. International Ladies' Garment Workers' Union, 1710 Broadway, New York, N.Y. 10019. National Board of the Coat and Suit Industry, 450 Seventh Ave., New York, N.Y. 10001. National Dress Manufacturers' Association, Inc., 570 Seventh Ave., New York, N.Y. 10018.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
C. Three-Dimensional Figures	C. 1. Define a polyhedron as an enclosed region of space which is bounded by many planes. 2. Define a dihedral angle as the union of a line and two noncoplanar half-planes having the line as edge. 3. Make a three-dimensional drawing of each "Platonic-Solid" (tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron).	designers earn from $85 to $125 a week. As the designer gains experience and is given more responsibility, their salary may range from $8,000 to $30,000 a year. Teaching Activity Suppose you are an apparel designer and you have been given the task of reducing the size of an original design which was placed on a sheet of paper 24" by 36". You are told that this design needs to be in a book with pages which measure 12" by 18". By using a pantograph (parallel ruler) with the proper setting you can easily reduce the size by simply tracing the original design.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
C. 1. Ask each student to list as many types of polyhedrons as he knows the name for: examples; pyramid, prism, etc. It may also be useful here to define what is meant by polyhedral region, face, edge, vertex, polyhedral angle, etc. 2. Define again a dihedral angle. Use an opened book as an example of a dihedral angle. 3. Ask each student to make a model out of construction paper or out of plastic for a tetrahedron, hexahedron, octahedron, dodecahedron, and an icosahedron. Then ask them to list the number of edges, faces, and vertices in each.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XI. Triangles and Associated Relations	XI. THE STUDENT SHOULD BE ABLE TO:	Concept Relationship of finding the area of a triangle to the job of a sail-lay-out man. Performance Objective Calculate the area of canvas needed by a sail-lay-out man to make a sail when given the linear dimensions. General Information Students wishing extra activities should be encouraged to research other careers in the sail boat industry. They may wish to compile and analyze material on work done on jobs concerning sailing. Examples: 1. Shipwright 3. Sail finisher 2. Sail cutter 4. Sailmaker High school students wishing to be involved in sailing should enter an apprenticeship program after graduation. Teaching Activity A sail-lay-out man calculates dimensions and specifications of sails from sales orders and enters data on worksheets for the sail cutter. A sail-lay-out man has received an order for a sail which is to be made from an expensive canvas material. Since he can order the canvas in precut sizes and shapes he needs to be accurate in his calculations in order to reduce cost. The sail he is constructing
A. Transversal Lines	A. Construct with 80% accuracy the division of a line segment into equal parts.	
B. Medians of a Triangle	B. When given the drawing of a triangle, construct the medians of the triangle.	
C. Median of a Trapezoid	C. Define the median of a trapezoid as the segment joining the midpoints of the legs of the trapezoid.	
D. Perpendicular Bisectors of a Triangle	D. Construct with 90% accuracy the perpendicular bisectors of the sides of a triangle.	
E. Bisectors of the Angles of a Triangle	E. Construct with 90% accuracy the angle bisectors of a triangle.	
F. Altitudes of a Triangle	F. Define the altitude of a triangle as the perpendicular segment from any vertex of the triangle to the line that contains the opposite side.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XI. A. Ask each student to draw several line segments and then construct the division of the segments into a designated number of congruent parts. B. Review constructing the median of a triangle. The median should go from a vertex to the midpoint of the opposite side. Ask each student to construct the three medians of three triangles and then check his work either with a ruler or compass. C. Ask each student to construct the median of several trapezoids. All he has to do is bisect the two non-parallel legs and connect the mid-points. The median should be parallel to both the bases. D. Ask each student to construct three perpendicular bisectors of each of three triangles. These lines should both bisect the sides and be perpen-dicular to the sides. If this construction is done accurately then the three bisectors should meet at a certain point inside the triangle. E. Ask each student to construct three bisectors of the angles of each of three triangles. Again this can be checked for accuracy by checking the place where the three angle bisectors meet. F. Ask each student to construct the altitudes of two triangles. Emphasize that the altitudes of a triangle may be an extension of the side of the triangle. The altitude should go from an angle perpendicular to the opposite side.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		is to be in the shape of a triangle with a height of 10 feet and a base of 6 feet. Solutions: $A = \frac{1}{2} bh$ $A = \frac{1}{2} \cdot 6 \cdot 10$ A = 30 square feet of canvas needed

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XII. Ratio, Proportion, and Proportional Line Segments A. Definitions 1. Ratio	XII. THE STUDENT SHOULD BE ABLE TO: A. 1. Define a ratio as a comparison between two sets.	Concept Relationship of the proportionality theorem to the job of the printer. Performance Objective Calculate the width of a card which is 6 inches long so that when folded in half, it will have the same shape as when it is unfolded. General Information Students wishing extra activities should be encouraged to research other careers in printing. They may wish to compile and analyze material on specific and related printing careers. Approximately 400,000 were employed in the printing industry in 1970. The average salaries for printers in 1970 ranged from $4.26 to $5.24 per hour. Some of the specific occupations involved were: 1. Bookbinders 2. Compositors 3. Electrotypers 4. Photoengravers 5. Pressmen 6. Sterotypers 7. Mailers 8. Linotype operators High school students wishing to enter the printing industry should follow a basic curriculum such as mathematics with a thorough knowledge of spelling, punctuation, and the fundamentals of grammar. An apprenticeship
2. Proportion	2. Define a proportion as two ratios which are equal.	
B. Proportional Theorems	B. When shown a mathematical representation of the proportional theorems, write in words each theorem with 80% accuracy. (examples: 1) $ad = bc$, 2) $a/c = b/d$, 3) $b/a = d/c$, 4) Equal Numerators or Denominators, 5) $a+b/b = c+d/d$ or $a+b/a = c+d/c$, 6) $a-b/b = c-d/d$ or $a-b/a = c-d/c$, 7) Series of Equal Ratios)	
C. Proportional Division of Line Segments 1. Commensurable Line Segments	C. 1. Define a commensurable line segment as one which is exactly divisible by the same unit of measure an integral number of times.	
2. Incommensurable Line Segments	2. Define an incommensurable line segment as one which is not divisible by the same unit of measure an integral number of times.	

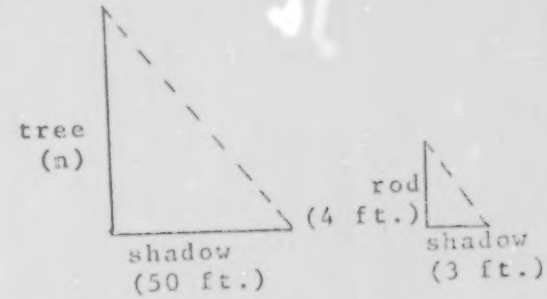
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XII. A. 1. Define ratio as a comparison of two numbers by division. Specify that the ratio is the ratio of the length of the two segments, two numbers, and not the ratio of the two segments themselves. Give four examples on the board. 2. Define a proportion as two ratios which are equal. Give some examples of proportions on the board and emphasize that a proportion is an algebraic equation. B. Derive each of the proportional theorems emphasizing that the derivation is purely algebraic. Review the students on any algebraic properties which they are not proficient. Assign several practice problems to insure that the students recall the algebra and understand the throrems. C. 1. Demonstrate the proportional division of a line segment by construction. Stress the correct positioning of the first of the parallel lines. Stress that it must intersect the end of the given line. 2. Define an incommensurable line segment and illustrate an example on the board.	Curriculum ESC-Region 20 Ratio and Proportion in Mathematics; 16mm film -- 4496 For Additional Information on Printing Careers American Newspaper Publishers Association, 750 Third Ave., New York, N.Y. 10017. Educational Council of the Graphic Arts Industry, Inc., 4615 Forbes Ave., Pittsburgh, Pa. 15213. Graphic Arts Technical Foundation, 4615 Forbes Ave., Pittsburgh, Pa. 15213. Gravure Technical Institute, 60 East 42nd St., New York, N.Y. 10020. International Typographical Union, P.O. Box 157, Colorodo Springs, Colo. 80901. Printing Industries of America, Inc., 1730 North Lynn St., Arlington, Va. 22209.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
D. Three-Dimensions	D. Prove the theorem with 80% accuracy which states the corresponding segments cut off on two lines which interest three parallel planes are proportional.	program lasting from 4 to 6 years is necessary after graduation. Apprenticeship may be aided by taking print shop offered by many high schools. Teaching Activity Suppose that you are the chief printer for the Blue Moon Printing Company. You have been requested to print a card 6 inches long, and wide enough so that when it is folded in half, it will have the same shape as when it is unfolded. What should the width of the card be, and how would you calculate it? Solution: Apply the proportionality theorem as follows: $6 : x :: x : 3$ $x^2 = 18$ $x = \sqrt{18} = 3\sqrt{2}$ $x = 4.24$ inches

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
D. Demonstrate the proof of the theorem and illustrate it with well drawn illustrations or a three dimensional model.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XIII. Similar Polygons A. Solutions	XIII. THE STUDENT SHOULD BE ABLE TO: A. When given two similar polygons, (one of the polygons has given measures) determine with 80% accuracy the measurements for the unmeasured polygon by using proportions.	Concept Relationship of the study of similar triangles to the specialized training needed by the forester. Performance Objective Calculate the height of a large tree by comparing the shadow of the tree to the shadow of a stake transit used by the forester. General Information Students wishing extra activities should be encouraged to research other careers in forestry. They may wish to compile and analyze material on work done in forestry. Examples: 1. Forestry aids or technicians 2. Range managers Approximately 14,700 were employed directly in forestry in 1970 (forest aid or range managers). Many foresters are employed in the heavily forested States of Washington, California, Oregon, Idaho, Utah, and Montana, as well as in the forested areas of the Great Lakes States, the Northeast, and the South. High school students wishing to enter forestry should qualify for beginning through work experience (in many cases a government sponsored training program). A bachelor's degree with a major in range management is required for the range manager.
B. Theorem	B. When given that two angles of one triangle are congruent to two angles of another triangle, make the statement that the triangles are similar.	
C. Special Proportions for Right Triangles	C. Define the geometric mean for any positive numbers a, b, and x, as x when $a/x = x/b$.	
D. Pythagorean Theorem	D. Prove the theorem with 90% accuracy which states that in any right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the length of the legs.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XIII. A. Review the definition of similar polygons pointing out that it states that corresponding sides are in proportion. Demonstrate that all linear parts of two similar polygons must then have the same ratio, that all angles are congruent, and that the ratio of the areas is the squares of the ratio of the sides. Work examples using lengths involving radicals and variables as well as integers. B. Review the theorem which states that the sum of the measures of the three angles of any triangle is 180°. Therefore, if two angles are fixed in size so is the third angle. Using a model demonstrate that once two angles are fixed the shape is fixed. C. Define geometric mean as $x = \sqrt{ab}$ and $a/x = x/b$. Explain that the mean is a number between a and b but not the average. Work a few examples and review the simplification of radicals. D. Demonstrate the proof of the pythagorean theorem. Emphasize that the theorem only holds true for right triangles. Work example problems showing how to find both legs and the hypotenuse. Review the students on squares, square roots, squaring radicals, and simplifying radicals. Also explain the converse of the pythagorean theorem.	Curriculum Harlandale Audio-Visual Center Pythagorean Triples; filmstrip -- AA-73 Similar Triangles; filmstrip -- Z-52 The Pythagorean Theorem; filmstrip -- Z-51 Career Harlandale Audio-Visual Center Forester; cassette tape -- cas. T-46 For Additional Information on Forestry Careers American Forest Institute, 1835 K St. NW., Washington, D.C. 20006. American Forestry Association, 919 17th St. NW., Washington, D.C. 20006. Society of American Foresters, 1010 16th St., NW., Washington, D.C. 20036. U.S. Department of Agriculture Forest Service, Washington, D.C. 20250.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
E. Special Right Triangles	E. 1. When given the length of the shorter leg of a 30^{o}-60^{o} right triangle, calculate with 80% accuracy the hypotenuse and longer leg on a written exercise. 2. When given the length of a leg of a 45^{o} right triangle, calculate the length of the hypotenuse on a written exercise.	Teaching Activity One of the major duties of a forester is to estimate standing timber and future growth. In performance of these duties he must often calculate the height of selected trees. By using his knowledge of similar triangles and a standard measuring instrument he can determine the height of the tree without climbing the tree. In calculating the height of the tree the forester places a measuring instrument (4 ft. tall) in direct sunlight perpendicular to the ground. He then notes that the measuring rod cast a shadow of 3 ft. on the ground. He then compares the shadow of the rod to that of the tree which he wishes to measure.  The forester realizes that two similar triangles are formed. He measures the length of the shadow of the tree (50 ft.). A proportion may be formulated between the two similar triangles. $4/3 = n/50$ $\quad 3n = 200$ $n = 66\ 2/3$ ft. tall

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
E. Demonstrate how the lengths of the sides of a 30°-60° right triangle can be calculated from an equilateral triangle and the pythagorean theorem. Emphasize $\sqrt{3}$, 2, $\sqrt{2}$, and how to use them in each triangle. Work a variety of problems and review the students on multiplying and dividing with radicals. Point out the 3-4-5 right triangle.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XIV. Area and Volume A. Area	XIV. THE STUDENT SHOULD BE ABLE TO: A. Given the measures of selected polygons, calculate the area of each polygon with 90% accuracy by using the correct formula.	Concept Relationship of the study of volume to the job of a die maker. Performance Objective Given the measurements of a paper weight which is in the shape of a pyramid, calculate the volume of material needed to make the paper weight. General Information Students wishing to obtain extra activities should be encouraged to research other careers in the machining industry. They may wish to compile and analyze material on specific machining careers. Examples: 1. All-round machinists 2. Machine tool operators 3. Tool and die makers In 1970 approximately 1.2 million machinists were employed in the United States. The national average salary for machinists in 1970 was between 4 and 5 dollars per hour. The increasing complexity of modern machinery and metalworking equipment is raising the technical requirements for tool and die making. A knowledge of mathematics, the basic sciences, electronics, and hydraulics will give young persons entering this occupation greater opportunities to advance their careers. Teaching Activity A die maker must be adapt in
B. Pythagorean Theorem	B. Given the measures of two sides of a right triangle, calculate with 90% accuracy the third side on a written exercise by using the pythagorean theorem.	
C. Hero's Formula	C. When given the measures of ten triangles, calculate their area with 80% accuracy by using "Hero's Formula" ($A=\sqrt{s(s-a)(s-b)(s-c)}$).	
D. Ratio	D. Solve by setting up a proportion with 80% accuracy for missing measures in polygons by using the theorem which states that "Every similarity between polygons with n sides has the property that the areas of the polygonal regions are proportional to the squares of the lengths of any pair of corresponding sides.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XIV. A. Derive the formula for the area of each required polygon. Where appropriate emphasize that the derivation simply adds the areas of convenient triangles to find the area of the polygon. Demonstrate that on some special polygons such as the equilateral triangle and square more than one formula may be used to determine the area. Remind the students that areas are measured in square units and be sure they know the measures of the commonly used polygons. B. Work examples of the pythagorean theorem and review the students on squares, square roots, squaring radicals, and simplifying radicals. C. "Hero's Formula" should probably be taught to the best students. Help them understand the derivation of the formula and demonstrate a typical problem. D. Review the students on squaring and taking the square root of fractions. Point out that the ratio of the areas, which are measured in square units, is the square of the ratio of the sides which are measured in linear units.	Curriculum ESC-Region 20 Possibly So, Pythagorus; 16mm film -- 8840 Career Harlandale Audio-Visual Center Your Future as a Tool and Die Maker; magnetic tape -- MT-273 For Additional Information on Tool and Die Making Careers International Association of Machinists and Aerospace Workers, 1300 Connecticut Avenue, N.W., Washington, D.C. 20036 National Machine Tool Builders' Association, 2139 Wisconsin Avenue, N.W., Washington, D.C. 20007. National Tool, Die & Precision Machining Association, 907 Public Square Building, Cleveland, Ohio 44113.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
E. Solids 1. Definitions 2. Equal Solids 3. Cavalieri's Theorem 4. Formulas	E. 1. Make a three-dimensional drawing of a prism, parallelopiped, truncated prism, and frustum of a pyramid. 2. Make a three-dimensional drawing of two prisms which have different linear measurements. 3. Demonstrate the validity of Cavalieri's Theorem by calculating the volume of five differently shaped solids with 80% accuracy which have the same height and the same base area. 4. Calculate with 80% accuracy the volume of prisms ($V = Bh$), pramids ($V = \frac{1}{3}Bh$), and spheres ($V = \frac{4}{3}\pi r^3$).	calculating the volume of dies he must cast. A die maker is given the task of making a die for a plastic paper weight. The designer requests that the paper weight be in the shape of a square pyramid and have a base length of 3 inches and a height of 4 inches. In the making of the die he must calculate the volume of the requested pyramid. From his knowledge of mathematics he knows that to find the volume of a square pyramid one must use the formula $V = 1/3\ s^2h$ where s is the length of the base and h is the height of the pyramid. Solution: $V = 1/3\ s^2h$ $V = 1/3\ 3^2 \cdot 4$ $V = 12$ cubic inches

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
E. 1&2. Demonstrate correct drawings of each figure. Point out that any lines that are parallel on the solid should be parallel on the diagram, and that any parts that would be obscured should be dashed. Having three dimensional models of each solid will help the student visualize them. 3. Illustrate Cavalieri's Theorem by the use of a stack of cards or similar objects. Show how the shape can change without the volume changing. 4. Show the derivation of each formula and work representative examples. Have three-dimensional models available to help the student visualize each solid. Point out how special triangles may be used in certain instances.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XV. Loci A. Definition	XV. THE STUDENT SHOULD BE ABLE TO: A. Define a locus as the set of all those points that satisfy a given condition.	Concept Relationship of the study of loci to the specialized training needed by the FBI agent. Performance Objective Given two sets of points relating to the position of an illegal radio, determine the location of the radio. General Information Students wishing extra activities should be encouraged to research other careers in police work. They may wish to compile and analyze material on specific police careers. Examples: 1. FBI agents (approximately 7,900 were employed in 1970 with salaries ranging from $10,869 to $23,000 yearly) 2. Police officers (approximately 330,000 full-time officers were employed in 1970 with salaries ranging from $8,500 for a new officer to $23,000 for such positions as chiefs or commissioners. 3. Guards and watchmen (approximately 200,000 were employed in 1970 with salaries ranging from $3,848 to $15,000 yearly) Because of specializatin in police work it should be emphasized that a high school student wishing to go into police work should take college prepartory courses. FBI agents are required to have a law degree.
B. Solutions	B. When given ten problems which ask for a description of a locus, give with 80% accuracy a diagram in which the figure representing the locus is clearly indicated and an accurate word description in which all specific details of the locus are mentioned.	
C. Intersection	C. When given ten problems which involve the intersection of loci, determine with 80% accuracy the intersection.	
D. Constructions	D. When given the necessary and sufficient conditions for ten problems concerning constructing loci, construct with 80% accuracy a diagram of the problem.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XV. A. Define a locus as the set of all those points that satisfy a given condition. Point out that in construction problems the students have been locating particular points or sets of points. It is now time to describe them accurately. B. Illustrate with drawings and models various loci and give the students examples of accurate word descriptions of the loci. Require the student to draw accurate diagrams and give accurate word descriptions of various loci. Emphasize the correct and accurate wording of the loci descriptions. C. Illustrate the intersection of loci with drawings and three-dimensional models as students may have difficulty visualizing the intersections. Also give examples of the correct descriptions of the intersections. Make a written assignment. D. Demonstrate the correct methods of constructing various loci reviewing the students on the basis of constructions if necessary. Assign the students a variety of loci constructions.	Curriculum Harlandale Audio-Visual Center Locus; 16mm film -- 16-548 Locus; filmstrip -- X-37 For Additional Information on FBI Careers The Federal Bureau of Investigation, U.S. Department of Justice, Washington, D.C. 20535.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		Teaching Activity An FBI agent has been called in by a local police department to locate and shut down the operation of an illegal radio transmitter which is broadcasting in a four state area. After preliminary investigation the agent determines the general location of the transmitter in a large metropolitan area. He then uses detection devises to locate the strength of the signal. From the strength of the signal received at the agent's sight, the distance between that point and the source is estimated at 3/4 of a mile. From this information the agent begins to pin-point the location. First of all the locus of all possible points on the 3/4 mile circle are plotted. Additional information received shows that the radio is one block west of a river in the area. Plotting the locus of all possible positions gives two points of intersection with the circle (points A and B). Since A is a vacant block, B is raided and the operation of the radio is halted.

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XVI. Properties and Relations for the Circle A. Definitions	XVI. THE STUDENT SHOULD BE ABLE TO: A. 1. Define an arc as a part of a circle. 2. Define a central angle as an angle formed by the intersection of two radii of the same circle. 3. Define a radius as a segment joining the center of a circle to a point on the circle. 4. Define a chord as a segment having its endpoints on a circle. 5. Define a diameter as a chord which contains the center of a circle. 6. Define concentric circles as two or more circles that lie in the same plane and have the same center.	Concept Relationship of concentric circles to the job of the botanist. Performance Objective When given the cross-section of a tree trunk with clearly detectable growth rings, determine the approximate age of the tree by counting the rings. General Information Students wishing extra activities should be encouraged to research other careers relating to the life science field. They may wish to compile and analyze material on work done by the life science field. Examples: Botanists — Ecologists Zoologists — Embryologists Microbiologists — Geneticists Agronomists — Horticulturists Anatomists — Husbandry Biochemists — Nutritionists Biological Oceanographer — Pathologists Biophysicists — Pharmacologists Physiologists Approximately 180,000 persons were employed in the life sciences in 1970. Salaries ranged from $6,548 to more than $26,100. High school students wishing a career in life science should acquire all available science
B. Arcs and Central Angles	B. Prove with 80% accuracy the following theorems: 1. If in the same circle or congruent circles two central angles are congruent, their arcs are congruent. 2. If in the same circle or congruent circles two minor arcs are congruent, their central angles are congruent. 3. In the same circle or in congruent circles congruent arcs have congruent chords.	
C. Chords	C. Prove with 80% accuracy the following theorems: 1. In the same circle or in congruent circles congruent chords are equally distant from the center. 2. In the same circle or in congruent circles congruent arcs have congruent chords.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XVI. A. Define each term and illustrate it by a drawing or a model. Emphasize that concentric circles must be in the same plane. Demonstrate with a model how two circles may have the same center and not be concentric. Point out that a diameter is the longest possible chord. B. Derive the theorems and illustrate with a model that congruent arcs must have congruent chords. Have the students solve proof problems involving these theorems. C. Derive these theorems and illustrate each with accurate drawings or models. Have the student work proof problems involving these theorems and algebraic problems finding the lengths of chords or their distances from the centers of the circles. Be sure the students remember the pythagorean theorem and how it is used in these problems.	Curriculum Harlandale Audio-Visual Center Vocabulary Circles, I; filmstrip -- X-33 . Vocabulary Circles, II; filmstrip -- X-34 For Additional Information on Biological Careers American Institute of Biological Sciences, 3900 Wisconsin Ave. NW., Washington, D.C. 20016. Botanical Society of America, Inc., c/o Department of Botant, Rutgers University, New Brunswick, New Jersey 08903. Ecological Society of America, Connecticut College, New London, Connecticut 06320.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
D. Tangents	D. Prove with 80% accuracy the following theorems: 1. A tangent to a circle is perpendicular to the radius drawn to the point of tangency. 2. A line in the plane of a circle and perpendicular to a radius at its outer endpoint is tangent to the circle.	courses with strong emphasis on math. Students wishing a career in life science should obtain an advanced degree (possibly a Ph.D.) in their particular field of interest. A bachelor's degree may be adequate for some positions, but opportunities for promotion are few without graduate training. Teaching Activity The dicotyledonous tree annually forms a concentric layer or ring of wood on its truck or stem. These rings sometimes vary in thickness according to existing conditions for a particular year. The botanist can tell a great deal about the history of a particular forest by examining the concentric rings of a cross-section of a tree. He can also make a fairly accurate age estimate of the tree by counting the growth rings of a cross-section of the trunk.
E. Measurement	E. Prove with 80% accuracy the theorem which states that the measure of an inscribed angle is equal to half the measure of its intercepted arc.	
F. Inscribed Angles	F. Prove with 80% accuracy the following theorems: 1. The angle inscribed in a semicircle is a right angle. 2. Inscribed angles intercepting the same arc or equal arcs of a circle are equal.	
G. Construction	G. Construct with 90% accuracy the mean proportional to two given line segments.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
D. Define a tangent emphasizing that it must lie in the same plane as the circle Ask the students what angle a tangent forms with a radius to the point of tangency. Then explain that these theorems simply deal with this obvious relationship. Emphasize the differences and different usage of each theorem. E. Demonstrate the proof of the theorem. Remind the students that the measure of an exterior angle is equal to the sum of the two remote interior angles. Review the students on the measure of a central angle. F. Have the students discover that since an angle inscribed in a semicircle intercepts a semicircle, its measure must be one half of 180° or 90°. Use leading questions to guide the students through this discovery if necessary. Point out that inscribing an angle in a semicircle is a convenient way to form a right angle. G. Demonstrate the construction emphasizing that the required segments is located between the two given segments. Also emphasize that the justification is based on an angle inscribed in a semicircle being a right angle and remind the students that the altitude drawn to the hypotenuse of a right triangle is the geometric mean between the two segments of the hypotenuse. Have the students work the construction several times.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XVII. Measurements in a Circle A. Regular Polygons 1. Definitions	XVII. THE STUDENT SHOULD BE ABLE TO: A. 1. a. Define the radius of a regular polygon as the radius of a circumscribed circle. b. Define the apothem of a regular polygon as the radius of an inscribed circle. c. Define the central angle of a regular polygon as the central angle whose chord is a side of the polygon.	Concept Relationship of the study of central angles of a circle to the job of the illustrator. Performance Objective Given the numbers of people using a particular product, illustrate by the use of a circle the number of people using different brands of the same product. General Information Students wishing extra activities should be encouraged to research other careers in commercial art. They may wish to compile and analyze material on work done by the commercial artist. Employment of commercial artis through the 1970's is expected to increase slowly primarily as a result of the upward trend in business expenditures for visual advertising. Artistic ability and good taste are the most important qualifications for success in commercial art, but it is essential that these qualities be developed by specialized training in the techniques of commercial and applied art. In addition, education i the fine arts, -- painting sculpture, or architecture -- and in academic studies provides a good foundation for obtaining employment in commercial art and may be essential for promotion. Salaries vary widely according to ability and experience.
2. Formulas	2. a. Calculate with 80% accuracy the central angles of regular polygons on a written exercise by using the formula which states the central angle = $1/n\ 360^{\circ}$.	
	b. Calculate with 80% accuracy the perimeters of regular polygons on a written exercise by using the formula which states $p = s$ where p = perimeter and s = length of a side.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XVII. A. 1. Define each term and illustrate with a drawing or model. Stress the difference between the apothem and the radius, that the radius is longer, and that the apothem must be perpendicular to the side. Illustrate with a drawing that the radii of a regular hexagon form equilateral triangles and that an apothem, radius, and side form a 30°-60° right triangle. Also illustrate that the radius and apothems of a square form 45°-45° right triangles and of an equilateral triangle form 30°-60° right triangles. Have the students use these properties to determine the lengths of the various parts of regular polygons. 2. a. Remind the students that the formula for the measure of a central angle of a regular polygon is the same as the formula for an exterior angle. Have the students determine the measure of central angles if given the number of sides and given the central angle determine the number of sides. b. Review the definition of perimeter and work a few example problems.	<u>For Additional Information on Illustrating Careers</u> National Art Education Association, National Education Association, 1201 16th St. NW., Washington, D.C. 20036.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CURRICULUM CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
	c. Calculate with 80% accuracy the area of regular polygons on a written exercise by using the formula which states $A = \frac{1}{2}ap$ where A = area and p = perimeter of the regular polygon.	
B. Circumference	B. Calculate with 80% accuracy the circumference of circles on a written exercise when given the radius or diameter by using the formulas $C = 2\pi r$ or $C = \pi d$ respectively.	
C. Value of π	C. Write a one or two page report on the history and transcendental nature of π.	
D. Area of a Circle	D. Calculate with 80% accuracy the area of circles on a written exercise by using the formula $A = \pi r^2$.	
E. Sectors and Segments 1. Definitions	E. 1. a. Define a sector of a circle as a region bounded by two radii and an arc of the circle. b. Define a segment of a circle as a region bounded by a chord and an arc of the circle.	Teaching Activity An advertising executive for a pharmaceutical firm has requested that a commercial artist produce an illustration for a particular aspirin sold by his firm. The commercial artist has been given the figures for the sales of major brands of aspirin for the past year. The artist must present an acceptable illustration of the pharmaceutical firm's product. Sales Information for Past Year Brand A - 160,000 units Brand B - 80,000 units Brand C - 60,000 units Brand D - 30,000 units Brand E - 20,000 units Brand F - 10,000 units The commercial artist decides to use a circle to make his illustration. His knowledge of the measurement of the central angle of a circle will aid in making his illustration.
2. Formulas	2. Calculate with 80% accuracy the areas of sectors and segments of circles using the appropriate formulas.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
c. Derive the formula at the sum of n triangles pointing out that the apothem of the polygon will be the altitude of the triangle. Demonstrate how the area can be determined for a regular hexagon, square, or equilateral triangle if only a side is known. B. Define circumference and review the definitions of radius and diameter. Introduce π as being the ratio of the circumference to the diameter. Explain that π is an irrational number and can only be approximated. Mention that the most commonly used approximations are 3.14 and 22/7. C. Ask each student to research and write a paper on pi. Ask the students to give their reports to the class. D. Derive the formula for the area of a circle from that of a regular polygon showing how the apothem becomes the radius and the perimeter becomes the circumference. Have the students work several problems using both approximations for pi. Explain that as with polgons the ratio of the areas of two circles is the square of the ratio of the radii, diameter, or circumferences. E. Define and illustrate both sector and segment. Explain that the formula for the area of a sector simply determines that portion of the area of the total circle, and that the area of a segment is the area of the sector minus the area of the triangle. Stress the logic behind these formulas to make them easier to memorize. Work several examples on the board and have the students work several problems.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XVIII. Mensuration for Solids with Curved Surfaces	XVIII. THE STUDENT SHOULD BE ABLE TO:	Concept Relationship of finding the volume of a sphere to the job of an oil pumper. Performance Objective When given the radius of a spherical oil tank, calculate its volume by using the formula $V = 4/3\ \pi r^3$. General Information Students wishing extra activities should be encouraged to research other careers in the oil industry. They may wish to compile and analyze material on work done by the petroleum industry. Examples: 1. Scientists - research 2. Drilling occupations 3. Maintenance workers 4. Pumpers In 1970 approximately 266,800 wage and salary workers were employed in the United States. Drilling is done for oil and gas in about three-fourths of the states, however, nearly 90 percent of the workers are employed in about 10 states. Most workers in non professional jobs with an exploration crew begin as helpers and advance into a more specialized job. In 1970 earnings of non supervisory employees in oil and gas extraction averaged $153.87 per week. Professional workers pay compared favorably
A. Cylindric Surface	A. Define a cylindric surface as a surface formed (traced or generated) by a straight line (the generator) that always moves parallel to a fixed line which intersects a fixed curve (the directrix) not in the same plane as the line.	
B.	B.	
1. Definition	1. Define a cylinder as a geometric solid formed by a closed cylindric surface and its intersection with two parallel planes.	
2. Inscribed Circumscribed	2. Make a three-dimensional drawing of an inscribed and circumscribed cylinder in regard to a prism.	
C. Right Circular Cylinders	C. Calculate with 80% accuracy the volume of right circular cylinders on a written exercise by using the formula $V = \pi r^2 h$.	
D. Conic Surface	D. Define a conic surface as a surface generated by a moving line (the generator) passing through a fixed point (the vertex) and touching a closed curve (directrix).	
E. Cone	E.	
1. Definition	1. Define a cone as a geometric solid bounded by one nappe of a conic surface between the vertex and a plane intersecting all elements of the nappe.	
2. Conic Sections	2. Make a three-dimensional drawing of conic sections formed by a circle, ellipse, parabola, and hyperbola.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XVIII. A. Define a cylindric surface. The student should not be held responsible for the definition but should understand the definition and be able to identify a cylindric solid. B. 1. If possible, use a model of a cylinder. Describe it first by pointing out that the bases are circles. After the description make a fromal definition. 2. Demonstrate three-dimensional drawings of cylinders pointing out that circular bases will appear as ellipses. C. Derive the formula for the volume of a right circular cylinder from that of a prism by showing that the area of the base will be πr^2. Work example problems and make a written assignment. D. Define a conic surface but do not hold the students responsible for the definition. E. 1. Using a model of a cone describe a cone. After the description make a formal definition. 2. If possible, use a model to show how conic sections form circles, ellipses, parabolas, and hyperbolas. Describe the different properties of each and illustrate with accurate drawings.	<u>For Additional Information on Petroleum Careers</u> American Association of Petroleum Geologists, P.O. Box 979, Tulsa, Oklahoma 74101. American Institute of Mining Metalurgical, and Petroleum Engineers (AIME), 345 East 47th Street, New York, N.Y. 10017. American Petroleum Institute, 1801 K St. N.W. Washington, D.C. 20006. National Petroleum Refiners Association, 1725 De Sales St. N.W., Washington, D.C. 20036.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
F. Right Circular Cone	F. Calculate the volume of right circular cones with 80% accuracy by using the formula $V = 1/3\ \pi r^2 h$.	with the pay of professional workers outside the oil industry. Teaching Activity An oil pumper must often prepare reports of the amount and quality of oil pumped and in storage. In the performance of his job he must often calculate the volume of holding tanks. One of the most modern forms of tanks for holding oil is the spherical shaped tank. The spherical tanks at one refinery have a radius of 30 feet. The oil pumper must calculate the volume of a tank in order to determine the holding capacity of the refinery. Solution: $V = 4/3\ \pi r^3$ $V = 4/3 \times 3.14 \times 30^3$ $V = 3{,}768$ cubic feet
G. Sphere 1. Formulas	G. 1. Calculate the volume of spheres with 80% accuracy by using the formula $V = 4/3\ \pi r^3$.	
2. Intersection	2. Define the intersection of a plane and a sphere as a small or great circle.	
3. Plane Tangent to a Sphere	3. Define a plane which is tangent to a sphere as a plane and a sphere which have only one point in common.	
H. Similar Cylinders and Cones	H. Define similar cylinders and cones as ones which have similar bases and lengths in the same ratio as the bases.	
I. Frustum of a Cone 1. Formulas	I. 1. Define a frustum of a cone as that part of a cone that is included between the plane of the base and a plane that is parallel to the base and intersects the edges.	
2. Exercises	2. Calculate the lateral surface area and volume with 80% accuracy of a frustum of cones by using the formula L.A. $= \pi l\ (r_1 + r_2)$ and $V = 1/3\ \pi h\ (r_1^2 + r_2^2 + r_1 r_2)$ respectively.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
F. Derive the formula for the volume of a right circular cone from the formula for the volume of a prism. Demonstrate the use of this formula and have the students work several problems. G. 1. Review Cavalieri's Principle and show the derivation of the formula of the volume of a sphere. Only the better students should be held responsible for this derivation. Work examples of typical problems and have the students work several problems. 2. Ask the students what the intersection of a plane and a sphere would be. Illustrate the intersection with a model and define a small and a great circle. 3. Illustrate a plane tangent to a sphere. Ask the students to describe the relationships and then to define a plane which is tangent to a sphere. H. Define similar figures as having the same shape but not necessarily the same size. Therefore, similar cylinders and cones would have similar bases and lengths in the same ratio as the bases. I. Working with frustums of cones should probably be restricted to only the best students. All students could be exposed to a definition and description of frustums.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XIX. Surface Geometry on the Sphere A. Definitions 1. Axis 2. Poles 3. Spherical Theorems 4. Polar Distance 5. Great Circle 6. Small Circle B. Spherical Theorems C. Spherical Angles	XIX. THE STUDENT SHOULD BE ABLE TO: A. 1. Define the axis of a circle of a sphere as the line through the center of a sphere perpendicular to the plane of the circle. 2. Define the poles of a circle of a sphere as the points on the sphere where the axis of a circle intersect the sphere. 3. Define spherical distance as the minor arc of the great circle passing through two points on the sphere. 4. Define polar distance of any point on a circle of a sphere as its spherical distance to the nearer pole. 5. Define a great circle as a section of a sphere made by a plane passing through the center. 6. Define a small circle as a section of a sphere made by a plane not passing through the center. B. Prove the following theorems with 80% accuracy: 1. All points on a circle of a sphere have equal spherical distances from a pole of the circle. 2. A point is the pole of a great circle passing through two points, not the ends of a diameter, if the point is a quadrant's distance from each of the two points 3. If a radius of a sphere is perpendicular to a plane at the outer end of the radius, the plane is tangent to the sphere. C. Define a spherical angle as an angle between two great circles.	Concept Relationship of the study of spherical geometry to the specialized training received by an airline pilot. Performance Objective Given the names of two cities between which an aircraft is to travel, state the shortest flight path which the pilot may determine. General Information Studnets wishing extra activities should be encouraged to research other careers in the airline industry. They may wish to compile and analyze material on work done by the airline industry. Examples: 1) Pilots and copilots (Approximately 27,00 employed in 1970 by scheduled airlines; 1,600 were employed in 1970 by supplemental airlines; and 2,500 were employed in 1970 by the Federal Government. Salaries [illegible] about $30,000 a year on domestic scheduled airlines and $37,000 a year on international operations.) 2) Flight engineers 3) Stewardesses - Specified qualifications (19 to 27 years old, 5 feet 2 inches to 5 feet 9 inches tall, weight not to exceed 140 pounds, and must be in

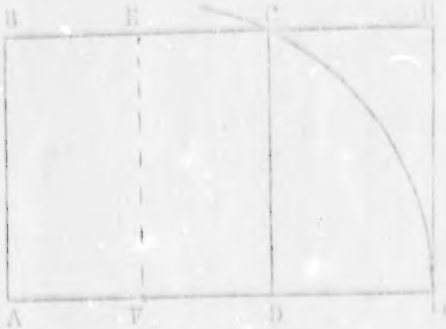
SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XIX. A. Use a model and a set of accurate drawings since students may have difficulty in understanding the parts of a sphere. First describe each term in easy to understand language before giving the correct definition in order that the student will clearly understand how the definition fits the term. Have the students identify each term on the illustration. B. Illustrate each theorem with a model and be sure the students understand what is being proved. Also remind them of the similar theorems stating that all points of a circle are equisdistant from the center and that if a line is perpendicular to a radius of a circle at its outer end the line is a tangent. C. Illustrate a spherical angle, define it, and point out its similarity to a central angle of a circle.	<u>For Additional Information on Air Transportation Careers</u> Addresses of individual companies are available from the Air Transport Association of America, 1000 Connecticut Ave. NW., Washington, D.C. 20036. Air Line Pilots Association, International, 1329 E. St., NW., Washington, D.C. 20004.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
D. Spherical Polygon 1. Definition 2. Theorem	D. 1. Define a spherical polygon as a figure on a sphere formed by the arcs of three or more great circles. 2. Prove the theorem with 80% accuracy which states that the sum of the sides of any spherical polygon (convex) is less than 360°.	excellent health) There are several thousand openings in this field each year. Salaries ranged in 1970 from $523 to $800 per month. 4) Mechanics 5) Airline dispatchers 6) Air traffic controllers 7) Ground radio operators 8) Traffic agents and clerks High school students wishing a career with the airlines industry must prove proficiency in their chosen field. A pilot must have a solid background in math with a degree (usually in math) from college. Traffic controllers and flight engineers also require a broad background in mathematics. Stewardesses, traffic agents, and clerks must master the proper use of language (foreign helpful). Teaching Activity In preparing or following a flight plan the pilot must consider the shortest route for the purpose of saving time and money. Since all airplane flights are made around the sphere of the earth, the pilot must be familiar with spherical geometry. In finding a shortest flight plan the pilot considers that the shortest distance between two points on a sphere is the arc of a great circle.
E. Spherical Triangles 1. Definition 2. Theorem	E. 1. Define a spherical triangle as a spherical polygon of three sides. 2. Prove the theorem with 80% accuracy which states that any two sides of a spherical triangle is greater than the third side.	
F. Polar Triangles	F. Make a three-dimensional drawing of a polar triangle.	
G. Spherical Triangles	G. Make a three-dimensional drawing of a spherical triangle which is composed of three right angles.	
H. Lune 1. Definition 2. Area	H. 1. Define a lune as a part of a sphere included between the halves of two great circles. 2. Calculate on a written exercise with 80% accuracy the area of lunes by using the theorem which states the area of a lune in spherical degrees is twice the angle of the lune.	
I. Area of Spherical Polygons	I. Calculate on a written exercise with 80% accuracy the area of spherical polygons by determining the excess of the sum of their angles over the sum of the angles of a plane polygon of the same number of sides.	
J. Zone of a Sphere	J. Make a three-dimensional drawing which illustrates the zone of a sphere.	
K. Spherical Segment	K. Define a spherical segment of one base as a segment where one of the parallel planes is tangent to the sphere.	

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
D. 1. Draw a spherical polygon on a sphere and emphasize that the arcs must be arcs of great circles. 2. Demonstrate the proof of the theorem and using a sphere illustrate how it must be true. Remind the students that the arcs must be arcs of great circles. E. Review the students on the theorem stating that any two sides of a triangle are greater than the third side. F. Illustrate a polar triangle on a sphere pointing out its special properties. Then show a three-dimensional drawing of a polar triangle. Require the students to reproduce the drawing. G. Draw a spherical triangle on a sphere then illustrate spherical triangles in three-dimensional drawings which indicate the angles at which the great circles will intersect. H. Preferably have a take apart model to illustrate a lune. Demonstrate how the area of a lune is calculated and work sample problems. I. Give the students the formula for the area of a spherical polygon and demonstrate a variety of problems on the board showing the relationship to plane polygons. Review the students on the areas of plane polygons if necessary. J. Define the zone of a sphere and describe it in less formal language with the use of a model. K. Using a model which disassembles illustrate a spherical segment and describe its relation to the segment of a circle.		

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
XX. Additional Topics A. Theorems of Pappus 1. Surface Theorem 2. Volume Theorem B. Golden Section C. Coordinate Geometry	XX. THE STUDENT SHOULD BE ABLE TO: A. 1. When given the length of a generating curve (L) and the distance of its center of gravity (X), calculate the surface area generated by the curve with 80% accuracy by using the formula $S = XL$. 2. When given the area of a generating curve (A) and the distance of its center of gravity (X), calculate the volume generated by the curve with 80% accuracy by using the formula $V = XA$. B. Bring a picture to class which illustrates the golden sections use in architecture. C. When given the coordinates of points on a coordinate plane, calculate with 80% accuracy the distance beteen two designated points on a written set of problems by using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.	Concept Relationship of the Golden Section (ratio) to the job of the architect. Performance Objective Given five architectural drawings, choose one of the five which illustrates the golden rectangle. General Information Students wishing extra activities should be encouraged to research other careers in architecture. They may wish to compile and analyze material on specific architectural careers. Examples: 1. Architect, construction 2. Architect, landscaping (salaries range from $7,000 to over $25,000 a year) A solid background in math is needed for the high school student wishing to go into architecture. Requirements for an architect generally require graduation from an accredited professional school followed by 3 years of practical experience in an architect's office. A license (acquired by test) is required by all states. Teaching Activity Show the students several examples of the golden rectangle as used in ancient architecture as well as modern architecture. Stress that the Golden Ratio is 1 : 1.618. Show the students the geometric construction of the

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS
XX. A. These topics should in most cases be used as extra work for the most advanced students. Therefore, the instruction should be on a more individual basis geared to the individual student's needs. The students should understand the derivation of the formulas as well as to be able to work problems using the formulas. B. A report on the golden sections use can be used as extra credit work for a student. C. Show the derivation of the distance formula stressing that it is simply an application of the pythagorean theorem. Work a few examples on the board and have the students work several problems. Review the students on simplifying radicals.	Curriculum Harlandale Audio-Visual Center Geometry in Art; filmstrip -- X-38 Career Harlandale Audio-Visual Center Mathematics in Architecture Series; color slides (22 in series) -- CS-1 For Additional Information on Architectural Careers National Architectural Accrediting Board, 521 Eighteenth Street, N.W., Washington, D.C. 20006. Society of American Registered Architects, 1821 Jefferson Place, N.W., Washington, D.C. 20036. The American Institute of Architects, 1785 Massachusetts Ave., N.W., Washington, D.C. 20036.	

CURRICULUM CONCEPTS	CURRICULUM PERFORMANCE OBJECTIVES	CAREER CONCEPTS, PERFORMANCE OBJECTIVES, GENERAL INFORMATION, AND TEACHING ACTIVITIES
		Golden Rectangle. The geometric construction of a Golden Rectangle begins with a square (ABCD), which is divided in two equal parts by the dotted line EF. Point F now serves as the center of a circle whose radius is the diagonal FC. An arc of the circle is drawn (CG) and the base line AD is extended to intersect it. This becomes the base of the rectangle. The new side DG is now drawn at right angles to the new base, with the line BA brought out to meet it. The resultant Golden Rectangle has an unusual property: if the original square is taken away, what remains will still be a Golden Rectangle. 

SUGGESTED TEACHING METHODS CAREER AND CURRICULUM	AUDIO-VISUAL AND RESOURCE MATERIALS	TEACHER COMMENTS

AUDIO-VISUAL SOURCE INFORMATION

TITLE	TYPE	SOURCE	TIME	COLOR OR B/W
Possibly So, Pythagorus	16mm	International Film Foundation	14 min.	C
Postulates Triangles & Circles	FS	Curriculum Films Inc.		C
Pythagorean Triples	FS	Eye Gate House		C
Ratio and Proportion in Mathematics	16mm	Coronet Films	11 min.	C
Segments and Polygons	FS	Eye Gate House		C
Similar Triangles	FS	Popular Science		C
Solving Inequalities	FS	Popular Science		C
The Parallel Postulate	FS	Popular Science		C
The Pythagorean Theorem	FS	Popular Science		C
Triangles-Types and Uses	16mm	Coronet Films	11 min.	C
Vocabulary Circles, I	FS	Curriculum Films Inc.		C
Vocabulary Circles, II	FS	Curriculum Films Inc.		C
Vocabulary Lines & Angles, I	FS	Curriculum Films Inc.		C
Vocabulary Lines & Angles, II	FS	Curriculum Films Inc.		C
Vocabulary Lines-Relationships	FS	Curriculum Films Inc.		C
Vocabulary Polygons	FS	Curriculum Films Inc.		C
Vocabulary Triangles	FS	Curriculum Films Inc.		C
Your Future as a Draftsman	magnetic tape	Guidance Associates		
Your Future as a Surveyor	magnetic tape	Guidance Associates		
Your Future as a Tool and Die Maker	magnetic tape	Guidance Associates		
Your Future in Engineering Technology	record FS	Guidance Associates		

AUDIO-VISUAL SOURCE INFORMATION

TITLE	TYPE	SOURCE	TIME	COLOR OR B/W
Bisecting Angles and Segments	FS	Eye Gate House		C
Civil Engineering	cassette tape	Educational Program Corporation		
Congruent Figures	FS	Eye Gate House		C
Congruent Triangles	FS	Popular Science		C
Equations & Inequalities	FS	Popular Science		C
Forester	cassette tape	Educational Program Corporation		
Geometry in Art	FS	Curriculum Films Inc.		C
Geometry-Inductive and Deductive Reasoning	16mm	Coronet Films	14 min.	C
Introducing Shapes, Lines and Angles	16mm	Coronet Films	11 min.	C
Introduction	FS	Curriculum Films Inc.		
Locus	16mm	McGraw-Hill Textfilms	13 min.	C
Locus	FS	Curriculum Films Inc.		C
Logic Deductive Reasoning	FS	Curriculum Films Inc.		C
Logic Definitions	FS	Curriculum Films Inc.		C
Logic Introduction	FS	Curriculum Films Inc.		C
Logic Mistakes in Thinking	FS	Curriculum Films Inc.		C
Lumberman	16mm	Encyclopedia Britanica Films	15 min.	C
Mathematics in Architecture Series	color slides	J. Weston Walch		C
Measuring in Astronomy-How Big, How Far	16mm	Film Associates of California	12 min.	C
Metalworking Series	record FS	Singer/SVE		C
Parallelograms and Their Properties	FS	Popular Science		C